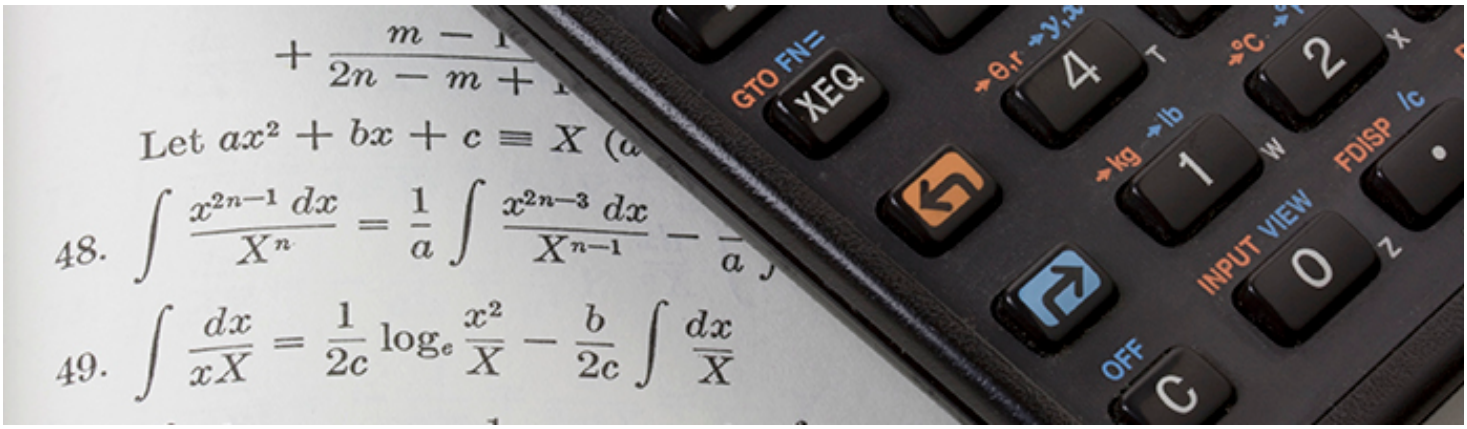
CAROLINA COURSES Online

THE UNIVERSITY OF NORTH CAROLINA @ CHAPEL HILL



MATH 381: Discrete Mathematics

WELCOME TO DISCRETE MATHEMATICS!

Since your first algebra course, you have been studying *continuous mathematics*, the mathematics based on the properties of the real number system. In this course, you return to the study of *discrete mathematics*, the mathematics based on the properties of the integers (the whole numbers). A large part of the mathematics you studied in grades K–8 was discrete mathematics, so in a way this is a return to familiar territory. However, if you think you know everything there is to know about the integers, you will soon be convinced otherwise!

In addition, this is a course in *the machinery of mathematics*, in particular the machinery of sets, mathematical logic, and proof. These matters were touched on, from time to time, in your calculus courses. Here, they will play an important role in everything we do.

You should expect to learn many things about familiar mathematical objects, and some not-so-familiar structures as well. You will learn some good techniques for problem solving. And I think you will find that discrete mathematics is, at its foundation, common-sense mathematics—something to be understood and enjoyed.

An introductory discrete mathematics course is designed to acquaint students with a number of topics. At first, this may look like a grab bag of unrelated material, but there are many interconnections and parallels among the

topics, and one of our goals is to illuminate these connections. Here are the topics we will consider:

1. **Sets and functions.** No doubt you have studied sets and functions before, but in this course, we will review the properties of these fundamental structures in a thorough and careful way.
2. **Propositional logic.** Mathematicians have developed some clear rules and useful notations for dealing with questions of logic. These rules will be essential to you in all subsequent mathematics courses.
3. **Counting.** You know how to count, but we will study techniques for counting the number of objects in a given set, such as the number of ways to rearrange the letters of Mississippi, or the number of different ways to select a dozen doughnuts if the shop has fourteen varieties. These techniques are important in many applications, and counting sets builds your problem-solving skills.
4. **Probability.** These days the basic ideas of probability are taught as early as the sixth grade, but with our counting techniques we will be able to compute the probabilities of intricate events.
5. **The integers.** We will explore several key properties of the integers related to prime numbers, divisibility, and factorization, and learn the techniques and uses of modular arithmetic, a subject in many advanced mathematics courses.
6. **Operations and relations.** Throughout the course, we will deal with a variety of operations and relations involving different mathematical objects. You will learn how to classify and describe these operations and relations so that you can recognize and analyze similar structures in other courses.

MATERIALS

See the course description for the most up-to-date list of materials.

HOW TO TAKE THIS COURSE

The course is divided into twelve lessons. Each lesson lasts from several days to two weeks (depending on content) and contains the following components:

- readings
- reading comprehension questions (ungraded)
- problems (graded)
- class discussion (graded).

The readings are are the equivalent of a classroom lecture. In each reading there are one or two **reading comprehension questions**. When you reach a question, **stop** and answer the question. If you can't answer it, go to the **discussion forum** (more information on this below) and see if the discussion there helps. If not, post a question there for the class.

When you finish each reading, send me an email with your answers to the reading comprehension questions. You won't be graded on these answers. I want to know whether you have understood the reading. If you do not understand the reading and are not able to get help from the discussion forum, send me your questions and I will respond as quickly as possible.

Each lesson also includes a set of problems to work and submit for grading. If you need help with the problems, post your questions on our discussion forum. You **cannot** ask for the answer to a problem, but you can ask how to begin, or what technique you should use. Everyone in the class, including me, is responsible for

coming up with answers to posted questions. If you don't have any trouble with the problems, you must participate in the discussion forum anyway. Help your classmates. If you can answer any of the posted questions, please do so. Or if you see a technique suggested for a problem and you can think of a better way to do the problem, please make your suggestion. **You will be graded on your participation in the discussions, and you must post three times per lesson,** either asking or answering questions, or sharing your comments and observations.

Submit the solutions to the problems on or before the last day of the lesson. (See the Schedule in the navigation bar at left for dates.) Instructions for submitting your assignments are in the first lesson.

The final exam for the course will count 30 percent toward your final grade. There are ten questions on the exam, and you will have several days to complete it. See the Schedule in the navigation bar at left for dates.

GUIDELINES FOR SUCCESS IN DISCRETE MATHEMATICS

Set a Routine and Stay on Course

- Assume everything will take twice the time you originally thought—here as with all things.
- Print the lesson pages and lecture notes if that helps you stay more organized.
- Do not get behind. Begin reading and working on the first day of each lesson so you can get and give timely help.
- Do not—ever!—leave the problems until the last day of the lesson. Start early.
- Set time aside each day—even just a few minutes if that is all you have—and check the discussion forums. Remember, you are responsible for helping your classmates with their questions if you can.
- If you don't understand something, ask for help right away.

Participate in the Discussion Forums

One of the most important aspects of any course is good and regular communication. Communication makes online courses real; lack of communication undermines them. Participating in an online course means making thoughtful contributions in the discussion forum—this is the only place where we are a class, a community of scholars. Frequent participation will help us all learn as much as possible from each other. You will be graded on your participation in the discussion forums. **You are required to post three times on three separate days per lesson,** but the general rule is: Let us hear from you often in the Forums.

Follow these rules:

1. Do not ask for the answer to an assigned problem. If you have difficulty with a problem, describe the difficulty and ask for a suggestion as to how you might proceed.
2. Do not give a classmate the answer to an assigned problem. To help, suggest a course of action—perhaps a theorem to consider or a way to structure the problem differently.
3. Keep the discussion cordial and make only helpful remarks (see Netiquette below). In our forums there is no cause for debate or criticism. Our purpose in communicating is to get help or give help, not to criticize.

If I don't routinely hear from you or "see" your presence on the discussion forums at least three times per lesson, I will be in touch to find out why you are not meeting your participation requirements. If you anticipate not being able to participate because of some personal or family emergency, let me know as far in advance as possible.

Ask Questions, Early and Often

The world's most talented instructors can still leave you uncertain and confused. Not only **can** confusion happen, it **will** happen. Mathematical text is not easy reading (you have probably noticed this before). In an online course there is no lecturer standing in front of you to stop and explain as the class proceeds through the material. So **ask**—whenever you need to, as often as you need to.

Who should you ask? First, check our discussion forums and see if others are having the same questions. Participate—ask and answer questions. Make it a general practice to post some question, answer, tip, or observation **every time** you get on your computer to work on the course. If you are not able to get help on the discussion forum, send me an email and I will respond as soon as possible.

Also, if at any time you feel lost or fall behind in the course, do not panic or keep silent—ask questions on the discussion forum so we can help, or send me an email. If you suffer silently, I may not know you're having difficulties until you have become discouraged—and I don't want that to happen!

NETIQUETTE

I am sure you know this, but I'll say it again for the record: It is important to be polite, respectful, and kind to your virtual classmates. Here are some general tips:

- **Keep your voice down**. USING ALL CAPITAL LETTERS is the equivalent of yelling. Ouch! Instead, use **bold**, *italics*, or *asterisks* around text to be emphasized.
- **Avoid sarcasm and subtleties**. Tone of voice and body language convey a substantial amount of information that is not available in written communications. It is easy to sound insensitive and hurt someone's feelings, or have them miss the point, when communicating through text only. To counter this, be clear and straightforward, and very careful with humor and joking.
- **Use Emoticons**. Some folks hate them, but these little icons can be quite effective in supplying primitive facial expressions to supplement your words. ;-) (See List of Common Emoticons.) If you don't like emoticons, use other ways of conveying your intentions such as the phrases "no offense meant," or "this is just my opinion."
- **Think before replying**. Assume any comment that sounds abrasive wasn't proofed for tone and wasn't meant to offend. If you are upset with a classmate, wait a little before posting a too-hasty reply that you may regret. Ask for clarification of the author's intent.

And last but not least, remember that your posts are recorded and saved in this course.

ACADEMIC POLICIES

By enrolling as a student in this course, you agree to abide by the University of North Carolina at Chapel Hill policies related to the acceptable use of online resources. Please consult the Acceptable Use Policy on topics such as copyright, net-etiquette, and privacy protection.

As part of this course, you may be asked to participate in online discussions or other online activities that may include personal information about you or other students in the course. Please be respectful of the rights and protection of other participants under the UNC-Chapel Hill Information Security Policies when participating in online classes.

When using online resources offered by organizations not affiliated with UNC-Chapel Hill, such as Google or YouTube, please note that the Terms and Conditions of these companies and not the University's Terms and Conditions apply. These third parties may offer different degrees of privacy protection and access rights to online content. You should be well aware of this when posting content to sites not managed by UNC-Chapel Hill.

When links to sites outside of the unc.edu domain are inserted in class discussions, please be mindful that clicking on sites not affiliated with UNC-Chapel Hill may pose a risk for your computer due to the possible presence of malware on such sites.

Academic Integrity

As a Carolina Courses Online student, you are responsible for obeying and supporting an honor system that prohibits lying, cheating, or stealing in relation to the academic practices of the University of North Carolina at Chapel Hill. **You are expected to do your own work in all aspects of your course.** The honor system also requires you to refrain from conduct that significantly impairs the welfare or the educational opportunities of others in the University community.

An especially serious Honor Code violation is plagiarism. Please view this brief Plagiarism Tutorial created by the librarians of UNC-Chapel Hill, Duke University, NC State University, and NC Central University.

Office of Accessibility/Special Accommodations

If you are a student with a documented disability, you can receive services through Accessibility Resources & Service. You must self-identify through Accessibility Resources to receive services or accommodation from either of these offices. Accessibility Resources works closely with programs, offices, and departments throughout the University to help create an accessible environment.

The office is located in Suite 2126 of the Student Academic Services Building (SASB), 450 Ridge Road, Chapel Hill, NC, and is open from 8 am to 5 pm Monday through Friday. You can contact them by phone at 919-962-8300 or 711 (NC-RELAY), or by email at accessibility@unc.edu.

LESSONS

Lesson 1: Combining Sets and Propositions Sets can be combined through union and intersection, operations that parallel the "and" and "or" operations in logic.
Lesson 2: Functions, Cardinality, and Probability Functions give us one way to compare the relative sizes of sets, and the basic ideas of probability give us another way.
Lesson 3: Implications, Quantifiers, and Proof We study implication propositions (such as theorems) and learn about direct and indirect proofs.
Lesson 4: Permutations and Combinations The basic rules for counting the number of elements in a set.

Lesson 5: The Binomial Theorem and Pascal's Triangle

The Binomial Theorem is one of the most important applications of the combination numbers C(n,r).

Lesson 6: Advanced Counting Techniques

Counting when selecting the same item more than once, or counting where there are identical items present.

Lesson 7: Sequences, Sums, and Recursive Proof

Recursive proof is often called mathematical induction—a powerful technique for studying discrete mathematics.

Lesson 8: Prime Numbers and Division

Prime numbers as the essential building blocks of the integers and the properties of multiplication and division.

Lesson 9: Remainders and Modular Arithmetic

Unlikely as it might seem, the study of remainders after division proves to be a powerful tool for understanding the integers.

Lesson 10: A Little Light Number Theory

The Euclidean Algorithm, one of the oldest theorems of number theory, and some other elementary theorems.

Lesson 11: Equivalence Relations

Cardinality and congruence are two examples of equivalence relations.

Lesson 12: Partial Orders

The subset and "divides" relations are examples of partial orders.